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A New Approach to the Problem of the Anomalous Magnetic Moment of the Electron

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Abstract

A heuristic model for deriving the anomalous magnetic moment of the electron is presented. A term $\alpha/2\pi - 0.327(\alpha/\pi)^2$ is deduced, in better agreement with experiment than is the QED derivation of $\alpha/2\pi - 0.328(\alpha/\pi)^2$. The result is strengthened by the recent non-QED account of the Lamb shift by Yu and Sachs.

The very recent explanation of the Lamb shift in helium by a theory which is fundamentally different from quantum electrodynamics (Yu and Sachs, 1975) adds impetus to the problem of reinterpreting the gyromagnetic anomaly of the electron. The Lamb shift in hydrogen has already been explained by similar self-consistent field theory (Sachs and Schwebel, 1961; Sachs, 1972). From these achievements it is evident that the theoretical interpretation of the Lamb shift is not necessarily a consequence of the assumptions of quantum electrodynamics. Even so, the great success of quantum electrodynamics in affording a very accurate estimation of the electron gyromagnetic ratio is indisputable, especially as it is augmented by the equally remarkable evaluation of the g factor of the muon. There are minor discrepancies which still trouble physicists (Bailey and Picasso, 1970), but the theory has proved versatile and adapted to meet the challenge of the very precise measurements. Even though today QED theory and measurement are a little out of line it would seem heretical to challenge the foundations of quantum electrodynamics in the present situation. In spite of this, a new approach is presented here. This new theory is less versatile but gives a better result for the electron g factor. It cannot provide an alternative to QED until it yields an account of the muon g factor. It does, meanwhile, offer an insight into a physical process which could approximate what is involved in the phenomenon.

The strength of this new explanation is its simplicity. The analysis is elementary. The weakness is that the physics involves a little speculation along unorthodox lines, though the method would have appeared orthodox before the advent of quantum mechanics. It is presented here since, though independent, it may complement the above-mentioned progress on the theory of the Lamb shift.

The electron of charge e is deemed to pervade a vacuum medium subjected to oscillations which radiate from the charge itself and which have a frequency mc^2/h, m being electron mass. The Coulomb field of the electron is deemed to be decoupled across a spherical boundary at what we may term the "resonant radius" r. This radius is centered on the mean position of the electron effective during the resonance period. A radial wave at the oscillation frequency will travel from the electron to the resonant radius and back to the electron in one natural period h/mc^2. Thus, for a point charge, r is half the Compton wave length.

For an electron in normal motion, that is one which is sensibly linear compared with the curvature implied by the resonant radius, the whole Coulomb field contributes to its effective mass. But if the electron has a nonlinear motion well contained within this curvature threshold it cannot carry all its field with it in its localized movement and its mass will be reduced by the effect of the decoupled field energy. The mass reduction δm is thus $e^2/2rc^2$ or, with r as $h/2mc$, simply e^2m/h. In terms of the fine structure constant α, δm becomes $(\alpha/2\pi)m$. This means that in this restricted state of motion, the so-called spin state, the electron exhibits a mass reduced by δm.

The gyromagnetic ratio for spin motion is generally expressed as e/mc, subject to QED adjustments. The g factor for the point charge electron is the ratio of e/mc as measured in spin to that observed for linear motion. Hence the g factor is simply $(1 - \delta)^{-1}$ or approximately $1 + \alpha/2\pi$. The QED derivation (Sommerfield, 1957) is $1 + \alpha/2\pi - 0.328\alpha^2/\pi^2$. Therefore we must now extend our analysis to higher order terms, allowing for the finite size of the electron.

If the electron has a finite spherical form it can be assigned a radius ka which defines the boundary from which the phase of its radial pulsations is set. This will displace the resonant radius by the same amount ka and so reduce the mass discrepancy slightly. We could then determine the radius of the electron from the experimental data of the gyromagnetic anomaly. It is found that there is agreement when the electron radius is approximately the accepted radius of the classical electron, that is approximately e^2/mc^2. Our problem is to pursue such analysis rigorously to see whether the gyromagnetic effect really can tell us something about the structure of the electron. We need not get involved in the problems of energy radiation. Indeed it is appropriate to keep in mind the fact that electron energy radiation is fraught with many difficulties. The problem of runaway solutions in the nonrelativistic Abraham–Lorentz equation of motion for the radiating particle is but one example (Daboul, 1974).

The most likely radius quantity applicable to the electron charge is that given by the formula $2e^2/3mc^2$. This appears in many classical works. It was favored by J. J. Thomson and owes its primary derivation to the link it affords with electromagnetic field energy (Born, 1965). It also corresponds to a particular charge distribution within the electron body which assures uniform electric field energy or pressure within the electron (Aspden, 1969). We therefore write:

$$a = 2e^2/3mc^2 \tag{1}$$

and, for generality, leave k as a factor relating the pulsation radius with the charge radius. This is done also in the knowledge that the electron g factor calculated will be too high if equation (1) is used directly as ka. We are instead guided by experiment to look for the justification for the k factor.

This leads us to a hypothesis. Whatever the true nature of the vacuum medium we know that the energy of the electron contained within the radius a is held in place by some action akin to pressure. It may be the pressure of a kind of gaseous medium or one due to repeated impact by photons. Our hypothesis is to suppose that outside the radius ka this pressure is random but that between the radii a and ka this pressure action involves radial ordering. Imagine some kind of corpuscle oscillating radially between radius a and radius ka. It has one degree of freedom. When in random motion outside radius ka it has three degrees of freedom. The uniformity of energy density within this pressure medium therefore implies a nonuniformity of pressure between the radius a and the limit radius ka. The corpuscles are all radially ordered along the electric field at radius a, making the pressure three times greater than it is outside radius ka in the region of random motion. The transition from radial to random motion may be gradual within this region between a and ka but there must be a distinct boundary at ka which acts as the surface for resonance of the radiated pulsations. Whatever the implications of such a hypothesis it sufficies to determine a value of k which we can use to deduce the electron g factor.

For a pressure $3P$ over a sphere of radius a to balance a pressure P over a sphere of radius ka due solely to radial pulsations the areas of the spheres must be inversely proportional to the pressures. Hence k is simply $3^{1/2}$.

The resonant radius of the finite electron becomes:

$$r = \frac{h}{2mc} + 3^{1/2}a \tag{2}$$

Since α is $2\pi e^2/hc$, we may write equation (1) as:

$$a = \frac{\alpha}{3\pi}\frac{h}{mc} \tag{3}$$

From (2) and (3):

$$r = \frac{h}{2mc}(1 + 2\alpha/3^{1/2}\pi) \tag{4}$$

The mass deficit for spin motion is then not $(\alpha/2\pi)m$, as calculated for the point electron but $(\alpha/2\pi)(1 + 2\alpha/3^{1/2}\pi)^{-1}m$. Writing this as δm, the electron

g factor becomes $(1-\delta)^{-1}$ or $1+\delta+\delta^2$ to second order. Substituting for δ this becomes:

$$g = 1 + \frac{\alpha}{2\pi} + [\tfrac{1}{4} - 3^{-1/2}]\left(\frac{\alpha}{\pi}\right)^2$$

$$= 1 + \frac{\alpha}{2\pi} - 0.327\left(\frac{\alpha}{\pi}\right)^2$$

With α^{-1} as 137.0359 this becomes 1.00115965, in close accord with the observed value of 1.0011596567(35) listed by Cohen and Taylor (1973). The established QED formulation gives a somewhat smaller value.

The author has not yet found any way of similarly explaining the muon g factor of 1.001165895(27) now measured (Cohen and Taylor, 1975). It is possibly relevant that the g factor for a point electron charge would, with second order terms included, be 1.00116276 and this happens to be remarkably close to the average g factor of the electron and the muon:

$$\tfrac{1}{2}(1.0011596567 + 1.001165895) = 1.001162776.$$

Note added in Proof. In developed versions of the Sommerfield formulation higher-order terms are added and these can assure the accuracy of the QED method, depending upon the value of α used.

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